values measured in an electric field of opposite polarity. It is evident from the functions G'(E) and G"(E) (Fig. 2) that the effect of the field depends significantly on the concentration of solid phase and increases with growth in C. This conclusion is illustrated clearly by the data of Fig. 3, which shows the log of the absolute value of the complex dynamic viscosity $|\eta^*|$ as a function of concentration C for various values of E. It is quite evident that without electric field action the value of $|\eta^*|$ increases monotonically with increase in C. However, depending on the electric field intensity, the rheological properties of such systems may change over a range of hundreds of thousands of times, with a significant role played by the concentration of the solid phase. In fact, from comparison of curves 1-6 (Fig. 3), it follows that the optimum electrorheological effect can be achieved only with consideration of both factors — the solid phase concentration and the value of the applied electric field. (At C = 20% this effect is significantly weaker than at C = 60%.)

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FLOW OF NON-NEWTONIAN LIQUID IN HELICAL CHANNELS WITH CONSTANT PITCH

E. K. Vachagina, R. S. Gainutdinov, and Yu. G. Nazmeev

UDC 532.517.2:532.135

We carry out an approximate variational solution of a problem of laminar steady-state flow of a nonlinearly viscous liquid with a formed velocity profile in helical channels.

Helical channels formed by placing helical inserts from strips twisted into a helix or screw inserts in a tube (Fig. 1) are an effective means of increasing convective heat exchange in non-Newtonian media [1].

Independently of the form of the transverse cross section, a helical channel has a helical symmetry. To use the available single-parametric symmetry group, we introduce new independent variables r', φ', and z' related to the independent variables of the cylindrical coordinate system by $r' = r$, $\varphi' = \varphi - (2\pi/S)z$, $z' = z$. The introduction of the new independent variables gives, by making the transformation of variables,

$$\frac{\partial}{\partial r}=\frac{\partial}{\partial r'},\quad \frac{\partial}{\partial \varphi}=\frac{\partial}{\partial \varphi'},\quad \frac{\partial}{\partial z}=-\frac{2\pi}{S}\frac{\partial}{\partial \varphi'}+\frac{\partial}{\partial z'}. \tag{1}$$

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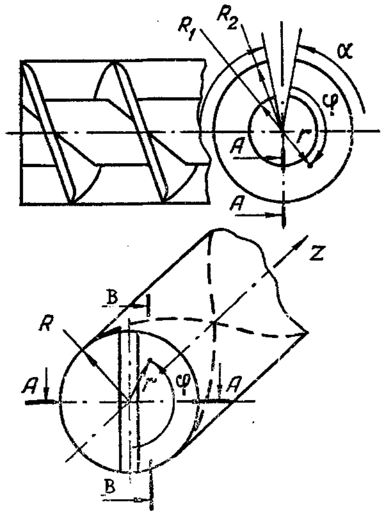

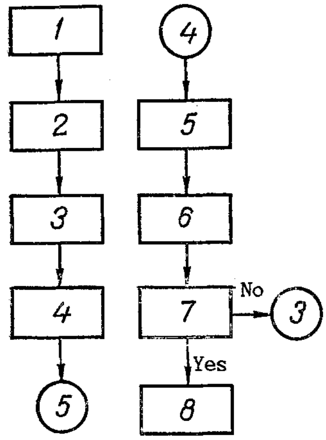

Fig. 1 Fig. 2

Fig. 1. Helical channels formed by strip and screw inserts.

Fig. 2. A schematic block diagram of the solution: 1) specification of the starting data; 2) μ is given the value μ_0; 3) calculation of ψ_{nm}, E_n; 4) solution of the system (17); 5) calculation of V_z, $\partial V_z/\partial\varphi'$, $\partial V_z/\partial r$; 6) calculation of μ; 7) estimate of the relative error; 8) printout of results.

In this case, fixed r' and φ' specify a helix, and ∂/∂z' is the derivative in the direction of the helical lines.

Since we are considering a flow with a formed velocity profile we shall assume that the operator ∂/∂z' with application to the velocity components is equal to zero. Then, under the condition that the forces of gravity are negligibly small, the system of equations of motion and continuity which describe the flow of nonlinearly viscous liquid in the helical channel with an arbitrary transverse cross section has the form (here and below, the primes of the new variable r' and z' will be omitted):

$$\rho\left(V_r\frac{\partial V_r}{\partial r}+\left(\frac{V_\varphi}{r}-\frac{2\pi}{S}V_z\right)\frac{\partial V_r}{\partial\varphi'}-\frac{V_\varphi^2}{r}\right)+\frac{\partial P}{\partial r}=$$

$$=2\frac{\partial}{\partial r}\left(\mu\frac{\partial V_r}{\partial r}\right)+\frac{1}{r^2}\left[1+\left(\frac{2\pi}{S}\right)^2r^2\right]\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_r}{\partial\varphi'}\right)+\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial(V_\varphi/r)}{\partial r}\right)+$$

$$+\frac{2\mu}{r}\left(r\frac{\partial(V_r/r)}{\partial r}-\frac{1}{r}\frac{\partial V_\varphi}{\partial\varphi'}\right)-\frac{2\pi}{S}\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_z}{\partial r}\right), \qquad (2)$$

$$\rho\left(\frac{V_r}{r}\frac{\partial(V_\varphi r)}{\partial r}+\left(\frac{V_\varphi}{r}-\frac{2\pi}{S}V_z\right)\frac{\partial V_\varphi}{\partial\varphi'}\right)+\frac{1}{r}\frac{\partial P}{\partial\varphi'}=$$

$$=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\mu r^3\frac{\partial(V_\varphi/r)}{\partial r}\right)+\frac{1}{r^2}\left[2+\left(\frac{2\pi}{S}\right)^2r^2\right]\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_\varphi}{\partial\varphi'}\right)+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V_r}{\partial\varphi'}\right)+$$

$$+\frac{2}{r^2}\frac{\partial}{\partial\varphi'}(\mu V_r)-\frac{2\pi}{S}\frac{1}{r}\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_z}{\partial\varphi'}\right), \qquad (3)$$

$$\rho\left(V_r\frac{\partial V_z}{\partial r}+\left(\frac{V_\varphi}{r}-\frac{2\pi}{S}V_z\right)\frac{\partial V_z}{\partial\varphi'}\right)-\frac{2\pi}{S}\frac{\partial P}{\partial\varphi'}+$$

$$+\frac{\partial P}{\partial z}=\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V_z}{\partial r}\right)+\frac{1}{r^2}\left[1+2\left(\frac{2\pi}{S}\right)^2r^2\right]\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_z}{\partial\varphi'}\right)- \qquad (4)$$

$$-\frac{2\pi}{S}\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r\frac{\partial V_r}{\partial\varphi'}\right)-\frac{2\pi}{S}\frac{1}{r}\frac{\partial}{\partial\varphi'}\left(\mu\frac{\partial V_\varphi}{\partial\varphi'}\right),$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rV_r)+\frac{\partial}{\partial\varphi'}\left(\frac{V_\varphi}{r}-\frac{2\pi}{S}V_z\right)=0 \qquad (5)$$

with the boundary condition

$$V_r|_\Gamma = V_\varphi|_\Gamma = V_z|_\Gamma = 0. \tag{6}$$

The second invariant of the velocity deformation tensor for the formulated problem has the form

$$I_2 = \left(\frac{\partial V_z}{\partial r}\right)^2 + \left(\frac{\partial V_\varphi}{\partial r} - \frac{V_\varphi}{r}\right)^2 + \frac{1}{r^2}\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left[\left(\frac{\partial V_z}{\partial \varphi'}\right)^2 + \left(\frac{\partial V_\varphi}{\partial \varphi'}\right)^2\right] + \frac{4}{r^2}\frac{\partial V_\varphi}{\partial \varphi'} V_r +$$
$$+ 2\frac{\partial V_r}{\partial \varphi'}\frac{\partial}{\partial r}\left(\frac{V_\varphi}{r} - \frac{2\pi}{S}V_z\right) + 3\left(\frac{\partial V_r}{\partial r}\right)^2 + 3\left(\frac{V_r}{r}\right)^2 + \frac{1}{r^2}\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left(\frac{\partial V_r}{\partial \varphi'}\right)^2. \tag{7}$$

The system (2)-(7) is a complete system of nonlinear equations of motion and continuity written in the absence of evolution terms with respect to z. An analysis of the formulated problem shows that a solution is possible in the following cases: 1) when constructing a complete iteration solution; this is possible for individual forms of transverse cross sections of helical channels, for example, a tube with a strip twisted into a helix; 2) when the form of the channel cross section is simplified; this is applicable for such forms as a tube with the screw insert where the cross section normal to the helical line is assumed to be a rectangle; 3) in the Stokes approximation of the problem using the "vortex-flow function" variables; 4) when using, as a solution, the first approximation of the complete iteration solution.

We consider more fully the last case. We assume that the direction of the flow velocity vector in a given point coincides with the direction of the helical line. This statement is equivalent to the assumption, in the first approximation of the complete iteration solution, that the radial component of the flow velocity is small. Results of experimental studies and generalizations show [2] that the assumption of small V_r on the whole does not distort the hydrodynamic picture which is observed in a helical channel.

Integrating the equation of continuity (5), in the absence of radial velocity component $\frac{\partial}{\partial \varphi'}\left(\frac{V_\varphi}{r} - \frac{2\pi}{S}V_z\right) = 0,$ with respect to φ' and using the boundary condition (6) gives a relationship between V_φ and V_z:

$$V_\varphi = \frac{2\pi}{S} r V_z. \tag{8}$$

Then, in the new variables, after the appropriate transformations, the system of equations which describes the first approximation of the iteration solution can be written in the form

$$\frac{1}{r}\frac{\partial P}{\partial \varphi'} = \frac{2\pi}{S}\frac{\partial}{\partial r}\left(\mu r \frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\frac{\partial}{\partial \varphi'}\left(\mu\frac{\partial V_\varphi}{\partial \varphi'}\right) + \mu\frac{4\pi}{S}\frac{\partial V_z}{\partial r}, \tag{9}$$

$$\frac{\partial P}{\partial z} - \frac{2\pi}{S}\frac{\partial P}{\partial \varphi'} = \frac{\partial}{\partial r}\left(\mu\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\frac{\partial}{\partial \varphi'}\left(\mu\frac{\partial V_z}{\partial \varphi'}\right) + \frac{\mu}{r}\frac{\partial V_z}{\partial r}, \tag{10}$$

$$V_\varphi = \frac{2\pi}{S} r V_z \tag{11}$$

with the boundary condition (6).

If we multiply Eq. (9) by $(2\pi/S)r$ and combine it with Eq. (10), we obtain

$$\frac{\partial P}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(\mu^* r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\frac{\partial}{\partial \varphi'}\left(\mu^*\frac{\partial V_z}{\partial \varphi'}\right), \tag{12}$$

where

$$\mu^* = \left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\mu; \qquad I_2 = \left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left\{\left(\frac{\partial V_z}{\partial r}\right)^2 + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\left(\frac{\partial V_z}{\partial \varphi'}\right)^2\right\}.$$

To find V_z and V_φ, we consider the system (11)-(12).

It is well known that there is no explicit solution of Eq. (12) for a general dependence $\mu(I_2)$, and similar problems in the hydromechanics of non-Newtonian liquids are solved by various numerical methods.

It follows from [3, 4] that the function $\mu(I_2)$ satisfies the following inequalities:

$$\mu(I_2) \geqslant c_1 = \text{const} > 0,$$
$$\mu(I_2) + 2\mu'(I_2) I_2 \geqslant c_2 = \text{const} > 0.$$

We consider the operator

$$LU = -\frac{1}{r}\frac{\partial}{\partial r}\left(\mu r \frac{\partial U}{\partial r}\right) - \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial}{\partial \varphi'}\left(\mu \frac{\partial U}{\partial \varphi'}\right).$$

We find the derivative of the operator L:

$$L'_u h = -\frac{1}{r}\frac{\partial}{\partial r}\left\{r\mu \frac{\partial h}{\partial r} + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left(\frac{\partial U}{\partial r}\frac{\partial h}{\partial r} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial U}{\partial \varphi'}\frac{\partial h}{\partial \varphi'}\right)\frac{\partial U}{\partial r}\right\} - \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial}{\partial \varphi'} \times$$
$$\times \left\{\mu \frac{\partial h}{\partial \varphi'} + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left(\frac{\partial U}{\partial r}\frac{\partial h}{\partial r} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial U}{\partial \varphi'}\frac{\partial h}{\partial \varphi'}\right)\frac{\partial U}{\partial \varphi'}\right\}. \tag{13}$$

It is seen from (13) that $L'_u h$ is a linear operator.

We show that $(L'_u h, h) \geq \nu\|h\|^2$, where ν is a positive constant:

$$(L'_u h, h) = \iint_\Omega -\frac{1}{r}\left\{\frac{\partial}{\partial r}\left(\mu r \frac{\partial h}{\partial r}\right) + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right] \times\right.$$
$$\left.\times \left(\frac{\partial U}{\partial r}\frac{\partial h}{\partial r} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial U}{\partial \varphi'}\frac{\partial h}{\partial \varphi'}\right)\frac{\partial U}{\partial r}\right\} h r dr d\varphi' -$$
$$-\iint_\Omega \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial}{\partial \varphi'}\left\{\mu \frac{\partial h}{\partial \varphi'} + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right] \times\right.$$
$$\left.\times \left(\frac{\partial U}{\partial r}\frac{\partial h}{\partial r} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial U}{\partial \varphi'}\frac{\partial h}{\partial \varphi'}\right)\frac{\partial U}{\partial \varphi'}\right\} h r dr d\varphi'.$$

Integrating by parts with allowance for the boundary condition $h|_\Gamma = 0$, we obtain

$$(L'_u h, h) = \iint_\Omega \left\{\mu\left(\frac{\partial h}{\partial r}\right)^2 + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\mu\left(\frac{\partial h}{\partial \varphi'}\right)^2 + \right.$$
$$\left. + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]\left(\frac{\partial h}{\partial r}\frac{\partial U}{\partial r} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right]\frac{\partial h}{\partial \varphi'}\frac{\partial U}{\partial \varphi'}\right)\right\} r dr d\varphi'.$$

We denote the vector $\operatorname{grad}^* h = \frac{\partial h}{\partial r}\bar{e}_1 + \sqrt{\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2}\,\frac{\partial h}{\partial \varphi'}\bar{e}_2$, then

$$(L'_u h, h) = \iint_\Omega \left\{\mu(\operatorname{grad}^* h)^2 + 2\mu'\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right](\operatorname{grad}^* h, \operatorname{grad}^* U)^2\right\} d\Omega.$$

Then, by analogy with [5], we write

$$(L_u'h,\ h) \geqslant c_3 \iint_\Omega (\mathrm{grad}^*\ h)^2\, d\Omega \geqslant c_3 \iint_\Omega (\mathrm{grad}\, h)^2\, d\Omega;\quad c_3 = \mathrm{const}.$$

Hence, using the Fridrichs inequality $(L'_u h,\ h) \geq \nu\|h\|^2$, the solution of the problem (12) is equivalent to finding a minimum of the functional

$$F(U) = \int_0^1 (L(tU),\ U)\, dt - \left(\frac{\partial P}{\partial z}, U\right);\quad U|_\Gamma = 0.$$

After transformations, the functional takes the form

$$F(V_z) = \iint_\Omega \frac{d\Omega}{\left[1 + \left(\frac{2\pi}{S}\right)^2 r^2\right]} \int_0^{I_2} \mu^*(\xi)\, d\xi + 2\frac{dP}{\partial z} \iint_\Omega V_z d\Omega. \tag{14}$$

To find the element which is responsible for the minimum of the functional (14), we use the Ritz method. This element will be sought in the form

$$V_z = \sum_1^m A_n f_n. \tag{15}$$

The coefficients A_n are found from the Ritz systems

$$\frac{\partial F(V_z)}{\partial A_n} = \iint_\Omega \left(\mu^*(I_2)\left(\frac{\partial V_z}{\partial r}\frac{\partial^2 V_z}{\partial r \partial A_n} + \left[\frac{1}{r^2} + \left(\frac{2\pi}{S}\right)^2\right] \times \frac{\partial V_z}{\partial \varphi'}\frac{\partial^2 V_z}{\partial \varphi' \partial A_n}\right) d\Omega + \frac{\partial P}{\partial z} \iint_\Omega \frac{\partial V_z}{\partial A_n} d\Omega = 0. \tag{16}$$

After the calculation of the integrands and appropriate transformations, system (16) can be written in the form of a system of nonlinear equations with respect to A_n:

$$\sum_1^m A_n \psi_{nm}(A_1,\ \ldots,\ A_m) + CE_n = 0, \tag{17}$$

where

$$C = \frac{\partial P}{\partial z} = \mathrm{const};\quad \psi_{nm} = \iint_\Omega \mu(I_2) f_{nm}(r,\ \varphi')\, d\Omega;\quad E_n = \iint_\Omega \frac{\partial V_z}{\partial A_n}\, d\Omega;\quad f_{nm}(r,\ \varphi')$$

is the result of evaluation of the expression in the round brackets of the first integral in (16).

To ensure stability of Ritz process [5] we choose, as the coordinate functions f_n, the eigenfunctions of a related operator

$$BU = \frac{\partial^2 U}{\partial \chi_1^2} + \frac{\partial^2 U}{\partial \chi_2^2} \tag{18}$$

for the regions under consideration.

The eigenfunctions of the operator B take the form: for a semicircle (tube with a strip insert)

$$f_n = C_{kp} J_k\left(\nu_{kp}\frac{r}{R}\right)\sin(k\varphi'),$$

where C_{kp} is chosen from the condition

$$\|f_n\|^2 = 1 = \frac{\pi}{2} C_{kp}^2 \int_0^R \frac{r}{R^2}\, dr \left(\nu_{kp}^2 J_k'^2\left(\nu_{kp}\frac{r}{R}\right) + \frac{R^2 k^2}{r^2} J_k^2\left(\nu_{kp}\frac{r}{R}\right)\right),$$

for a rectangle (transverse section of a tube with a screw insert)

$$f_n = \frac{2}{\pi}\left(\frac{k^2}{(R_2 - R_1)^2} + \frac{p^2(R_2 + R_1)^2}{4(\varphi_2 - \varphi_1)^2}\right)^{1/2} \sin\frac{k\pi(r - R_1)}{(R_2 - R_1)} \sin\frac{p\pi(\varphi' - \varphi_1)}{(\varphi_2 - \varphi_1)}.$$

By analogy with [3, 4], the system (16) was solved by the Gaussian method and, for the calculation of ψ_{nm} and E_n, we used a repeated Gaussian quadrature with eight nodes.

By way of concrete dependence $\mu(I_2)$, we use the generalized rheological Kutateladze–Khabakhpasheva equation for a non-Newtonian structurally viscous liquid [6] $d\varphi_* = -\Phi\beta d\tau_*$ in its most interesting particular form

$$\varphi_* = \exp(-\tau_*), \tag{19}$$

To solve system (11)-(12) reduced to the form (17) by using the rheological equation (19), we determine the order of the calculation. At the beginning of the calculation, the

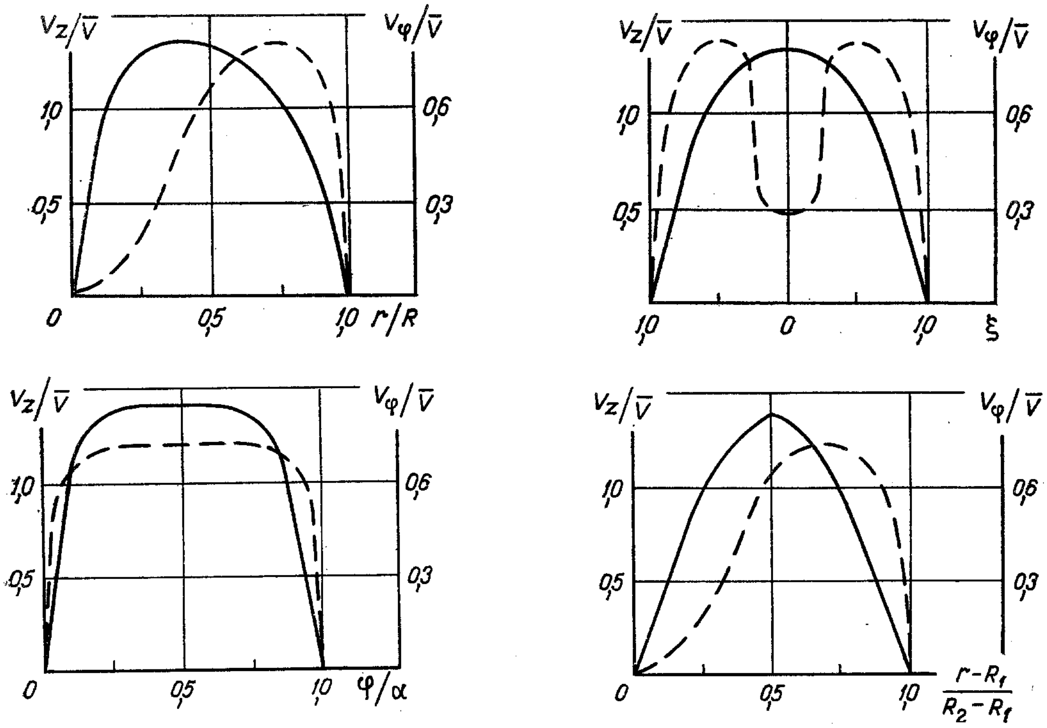

Fig. 3 Fig. 4

Fig. 3. Calculated dimensionless curves of the axial and circular velocity components (full and dashed lines, respectively) in the cross sections A–A and B–B ($r = R/3$). $\partial P/\partial z = 600$ N/m^3; $S = 0.08$ m; ξ is the dimensionless coordinate in the cross section B–B. Channel with a strip insertion.

Fig. 4. Calculated dimensionless curves of the axial and circular velocity components (full and dashed lines, respectively) in the cross sections $r = (R_1 + R_2)/2$ and A–A. Channel with a screw insertion. $\partial P/\partial z = 600$ N/m^3; $S = 0.08$ m.

where

$$\varphi = (\varphi_\infty - \Phi)/(\varphi_\infty - \varphi_0);\quad \tau_* = \theta\,(\tau - \tau_1)/(\varphi_\infty - \varphi_0);\quad \Phi = 1/\mu;\quad \tau = \mu\,(I_2)\sqrt{I_2}.$$

quantity $\mu(I_2)$ is given some value μ = const. It is clear that, for the model (19), this will be $\mu_0 = 1/\phi_0$. Thus, at the first iteration step, the coefficients μ in Eq. (12) are frozen. Calculating all values ψ_{nm} using a repeated Gaussian quadrature (solving the system (17)), we determine A_n in the first approximation. This is followed by the calculation of the matrix $\mu(I_2)$ using (19).

In the second iteration step, system (17) is solved again with allowance for the matrix $\mu(I_2)$ calculated in the first iteration step. The use of the matrix $\mu(I_2)$ from the first iteration step again freezes the coefficients in (12) and makes it possible to use the Gaussian method to determine A_n in the second approximation.

In the subsequent iterations, the procedure is repeated. If the sequence of solutions (A_{1k}, A_{2k}, A_{nk}) ($k = 1, 2, \ldots, \infty$) tends to a limit, this limit is the solution of the system (18).

A schematic block diagram of the formulated problem in the variational formulation is shown in Fig. 2.

To test the obtained solution, we calculated the hydrodynamic parameters for the case of a model structurally viscous liquid (solution Na-KMTs) with parameters $\theta = 0.2981$ [Pa$^2\cdot$sec]$^{-1}$; $\varphi_0 = 1.9$ [Pa·sec]$^{-1}$; $\varphi_\infty = 13.7$ [Pa·sec]$^{-1}$; $\tau_1 = 0$ in helical channels with the following geometrical dimensions: channel with strip insert: $R = 0.006$ m, $S = 0.08$ m = const; channel with a screw insert: $R_1 = 0.021$ m, $R_2 = 0.036$ m, $S = 0.08$ m = const.

Figure 3 shows the results of calculation of the velocity components in a channel with a strip insert. A comparison with the experiment [1] on the bulk flow rate shows the adequacy within the limits 15%. Figure 4 shows the analogous curves for a channel with a screw insert.

Previously, Nazmeev and Mumladze [7] solved this problem by using the iteration method of variable directions.

NOTATION

R, φ, z, polar coordinates; R', φ', z', new independent variables with helical symmetry; S, pitch; P, pressure; V_r, V_φ, V_z, radial, circular, and axial components of the flow velocity; μ, effective (structural) viscosity; Ω, the region; Γ, boundary of the region; F, functional subject to minimization; I_2, second invariant of the deformation velocity tensor; A_n, coefficients of the basis function; i, j, exponents of the power; Φ, fluidity of the non-Newtonian liquid; φ_0, φ_∞, fluidities for $\tau \to 0$ and $\tau \to \infty$; τ, intensity of the shear stress; θ, τ_1, measure and limit of the structural stability of the liquid; k, number of the iteration; L, an operator; U and h, arbitrary functions which satisfy the boundary conditions U, $h|_\Gamma = 0$; L_u', derivative of the operator L; $\bar{e}_1$, $\bar{e}_2$, unit vectors of the cylindrical coordinate system; β, exponent of the power in the rheological model; $(L_u'h, h)$, $(L(tU), U)$, $(\partial P/\partial z, U)$, scalar products in the space L_2; $\|h\|^2$, square of the norm of the element h in space L_2; R, radius of the tube; R_1 and R_2, radius of the inner surface of the outer tube and the radius of the outer surface of the inner tube; $\bar{V}$, mean-flow-rate velocity of the flow; f_n ($n = 1,\ldots,m$), a complete and linearly independent system of elements; B, a related operator; J_k, Bessel functions; and ν_{kp}, root of the Bessel function.

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AXISYMMETRIC PROBLEM ON THE IMPREGNATION OF A HEATED FILLER BY A VISCOPLASTIC LIQUID

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UDC 532.546

This article examines the penetration of a viscoplastic liquid (binder) into a preheated porous cylindrical braid (filler) moving inside it.

The study [1] examined a production process involving the continuous impregnation of porous fillers. This process is common in the manufacture of many composite materials. Since the viscosity of the binders is often too great at room temperature and since there are serious technical problems with the use of high pressure gradients, it has been proposed that fluid resistance during filtration be reduced by preheating the filler. A

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